

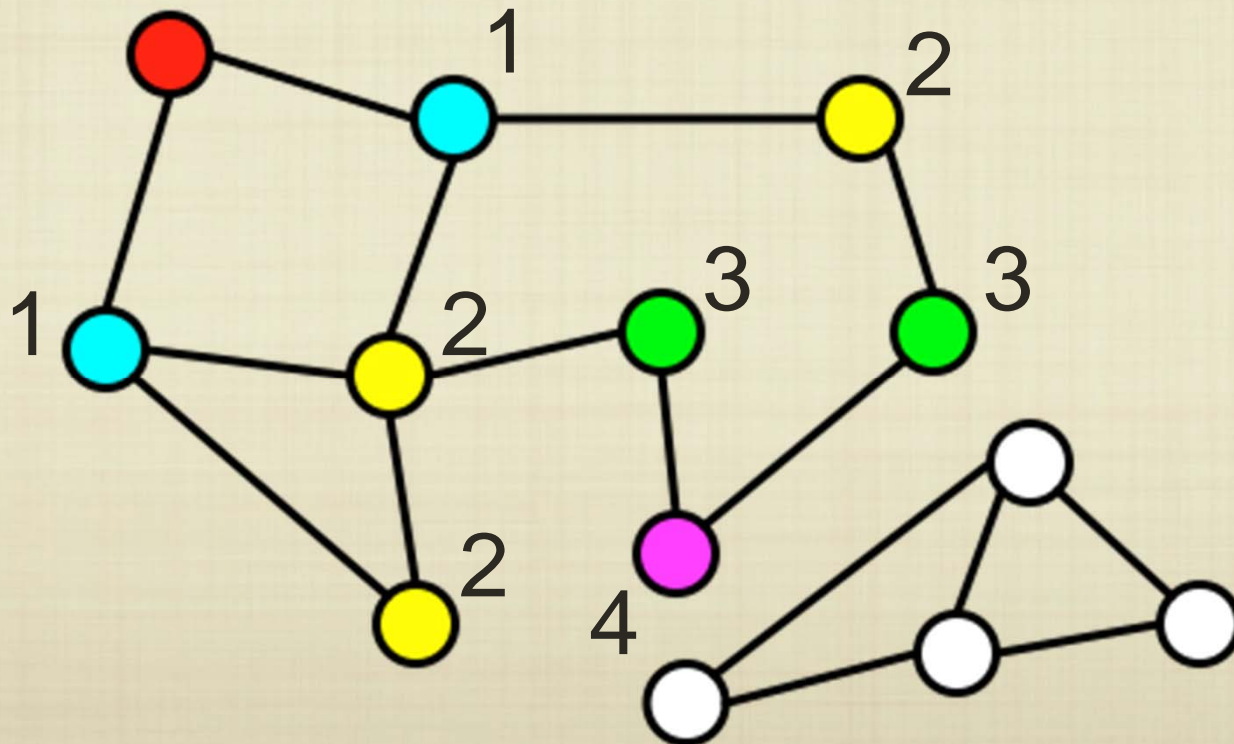
Introduction To Graphs and Networks II

Fall 2013
Carola Wenk

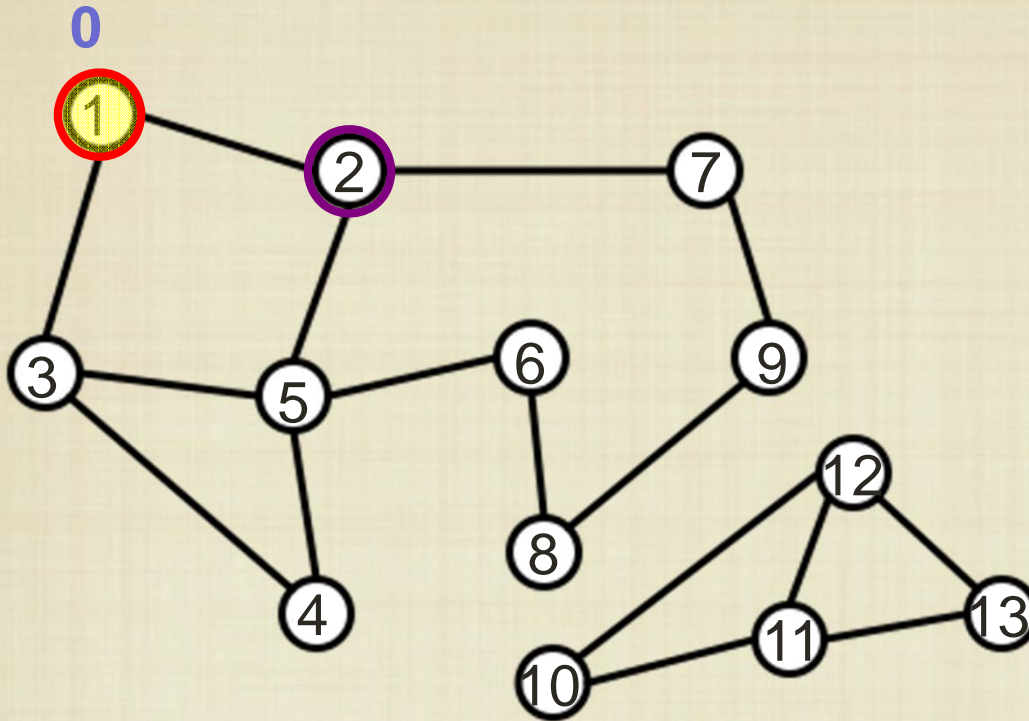
Breadth-First Search

Algorithm:

1. Initialize a queue with the starting vertex
2. While the queue is not empty
 - a. Dequeue the next vertex from the queue
 - b. Enqueue all its neighbors that have not been visited/enqueued before



Breadth-First Search



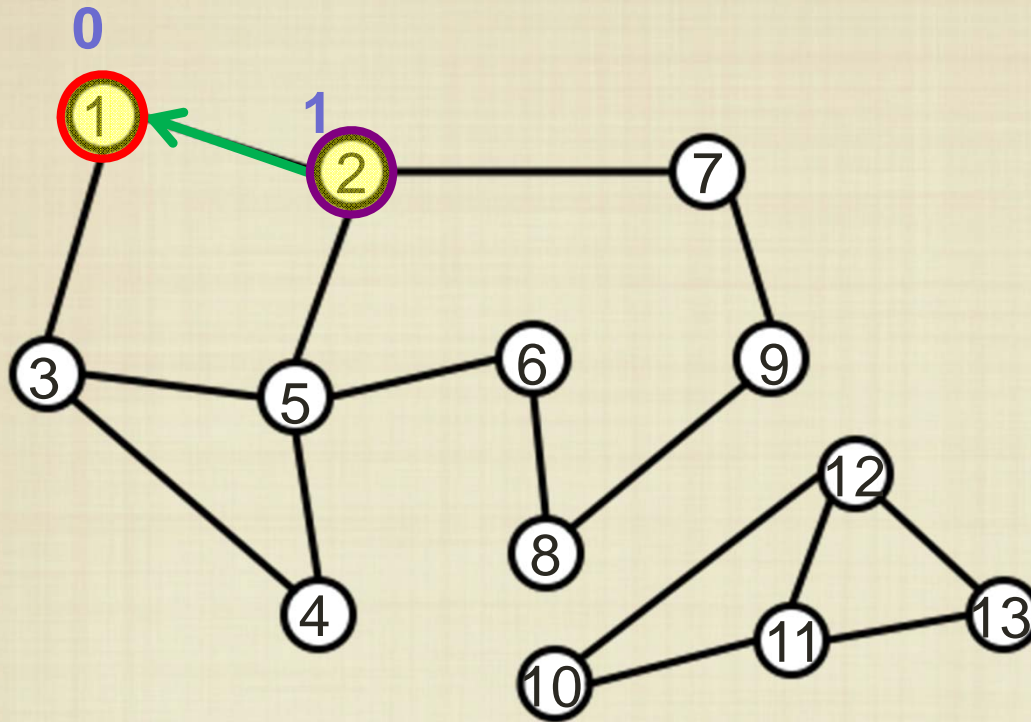
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 9:[7,8],
 10:[11, 12],
 11:[10,12,13],
 12:[10,11,13],
 13:[11,12]
 $\}$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	False	False	False	False	False	False	False	False	False	False	False	False
distance	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
pred	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

vertex = 1
i = 2

queue =

Breadth-First Search



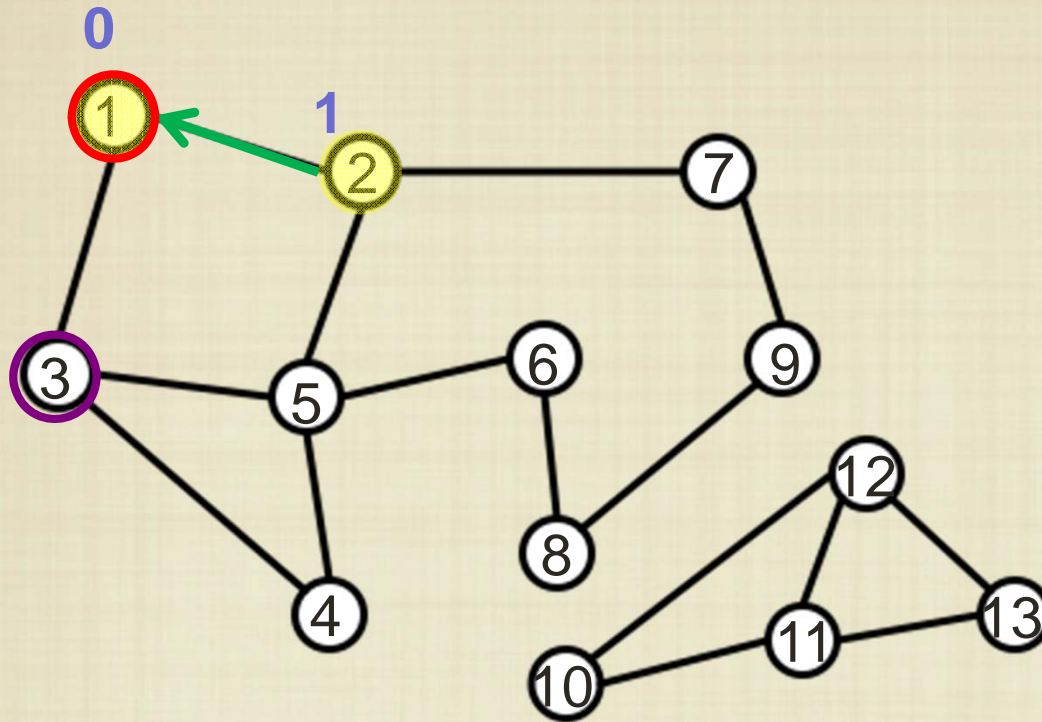
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pred	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

vertex = 1
i = 2

queue = 2

Breadth-First Search



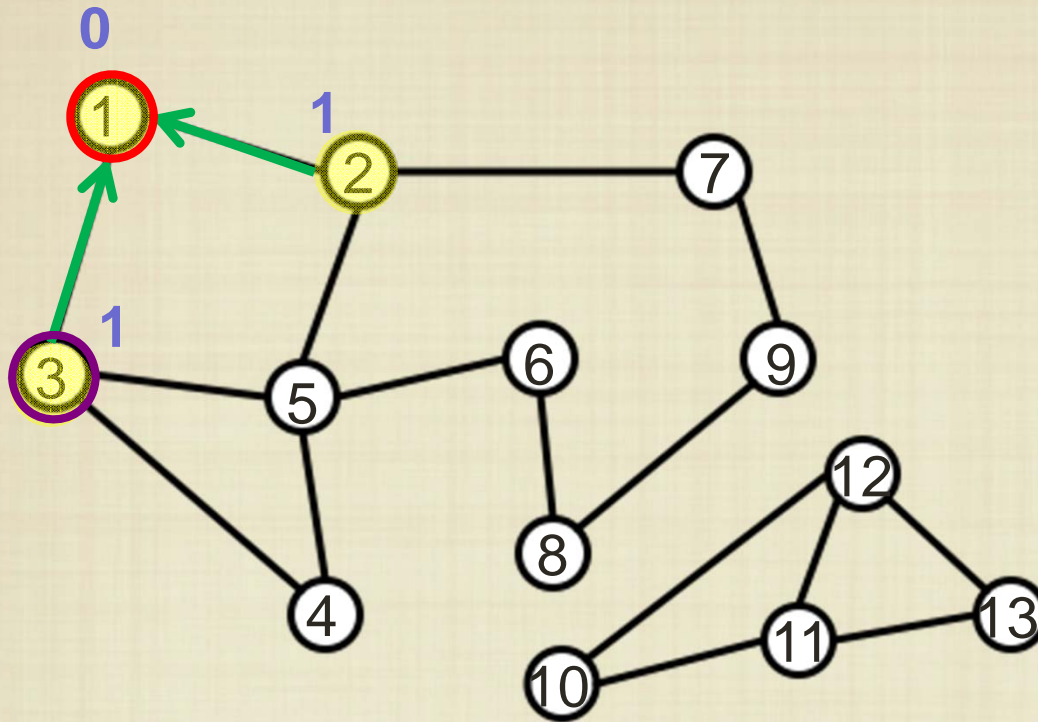
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pred	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

vertex = 1
i = 3

queue = 2

Breadth-First Search



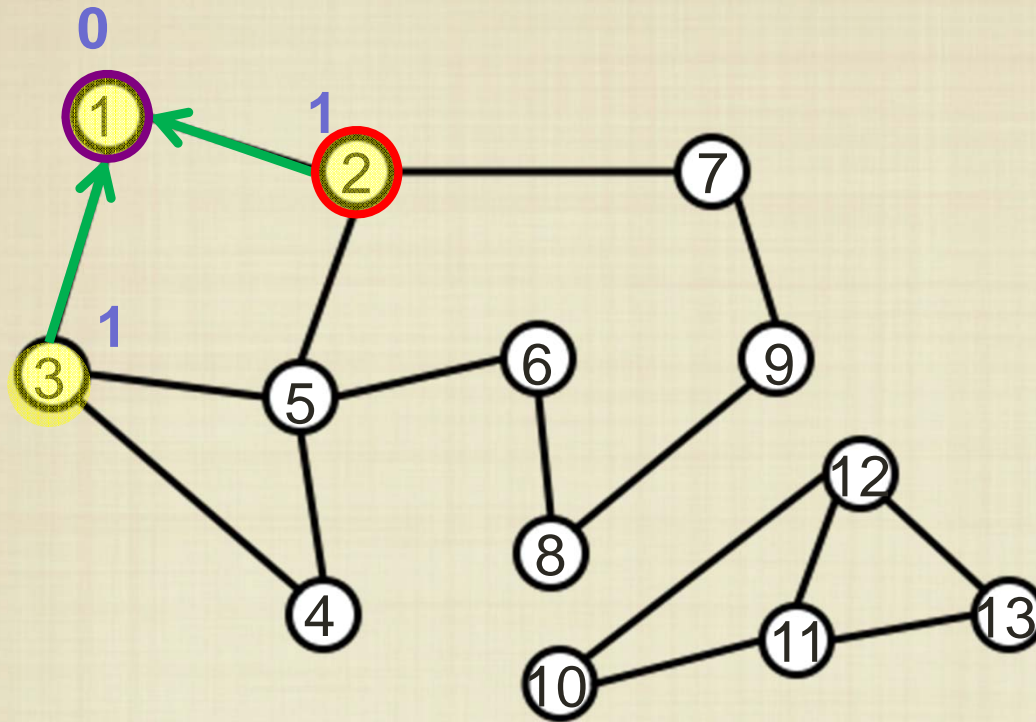
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pred	-1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

vertex = 1
i = 3

queue = 2, 3

Breadth-First Search



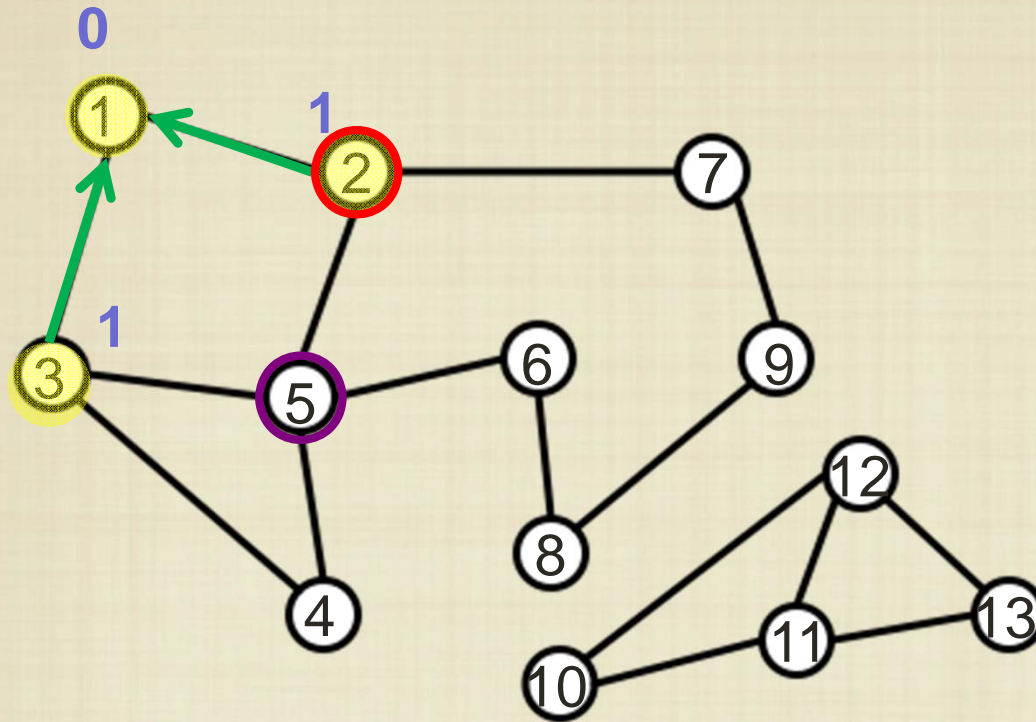
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pred	-1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

vertex = 2
i = 1

queue = 3

Breadth-First Search



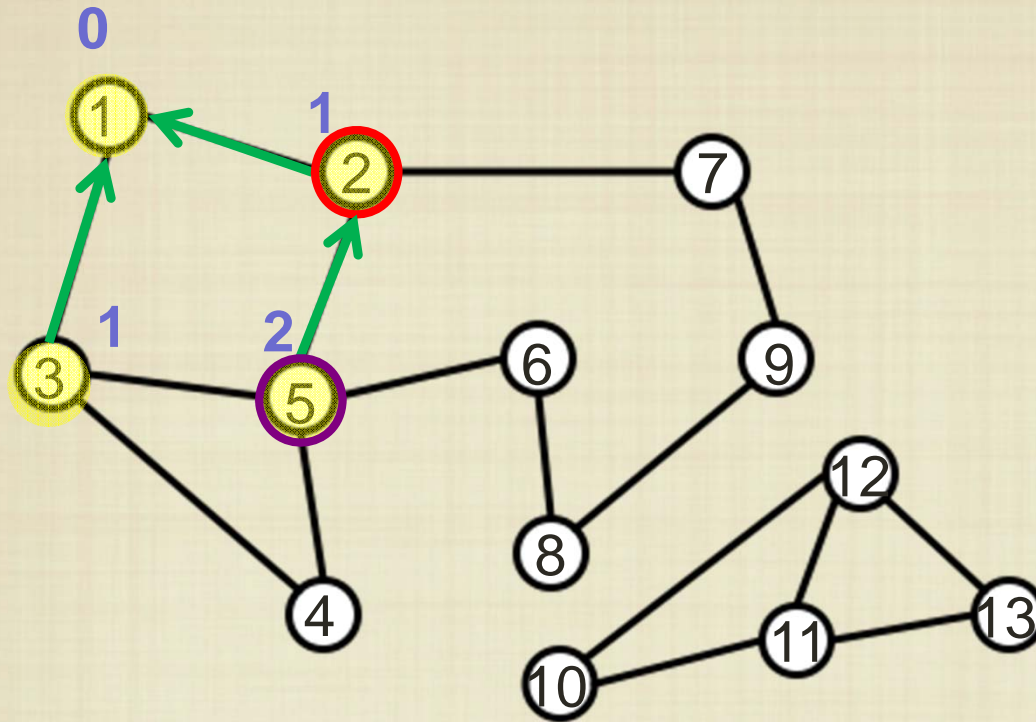
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distance	0	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
pred	-1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

vertex = 2
 i = 5

queue = 3

Breadth-First Search



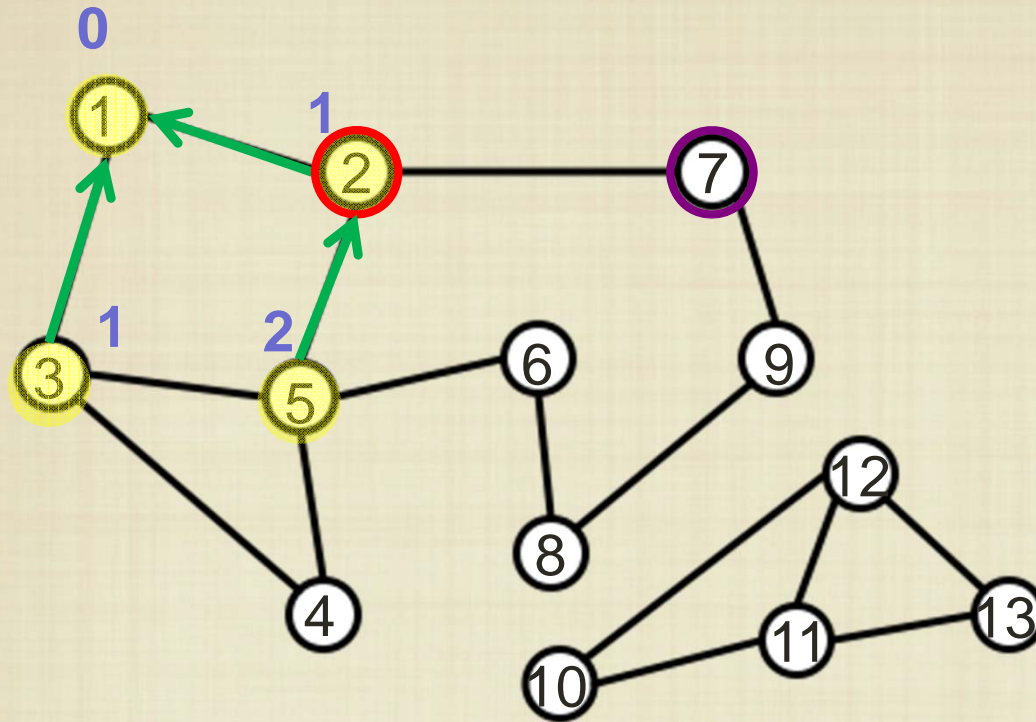
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distance	0	1	1	-1	2	-1	-1	-1	-1	-1	-1	-1	-1
pred	-1	1	1	-1	2	-1	-1	-1	-1	-1	-1	-1	-1

vertex = 2
i = 5

queue = 3, 5

Breadth-First Search



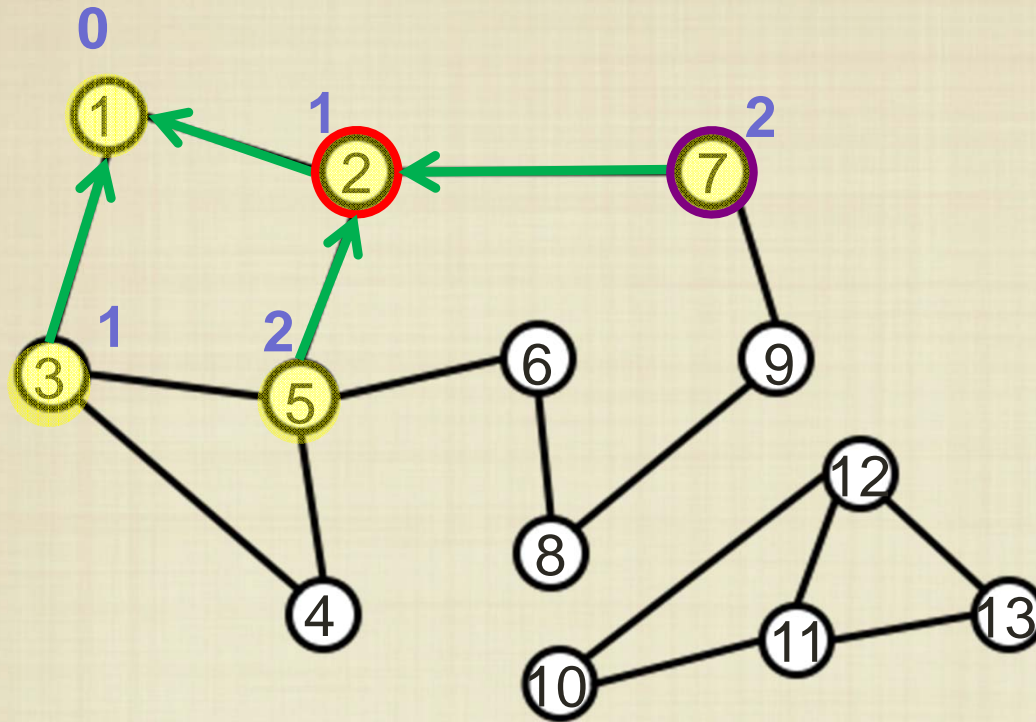
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pred	-1	1	1	-1	2	-1	-1	-1	-1	-1	-1	-1	-1

vertex = 2
i = 7

queue = 3, 5

Breadt-First Search



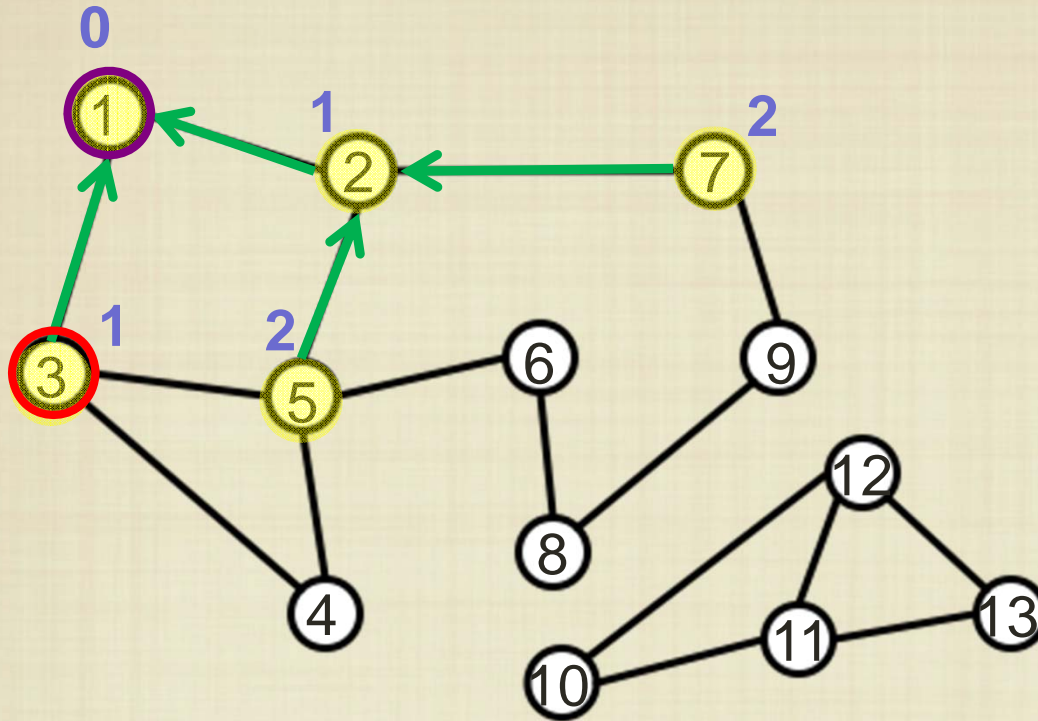
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pred	-1	1	1	-1	2	-1	2	-1	-1	-1	-1	-1	-1

vertex = 2
i = 7

queue = 3, 5, 7

Breadth-First Search



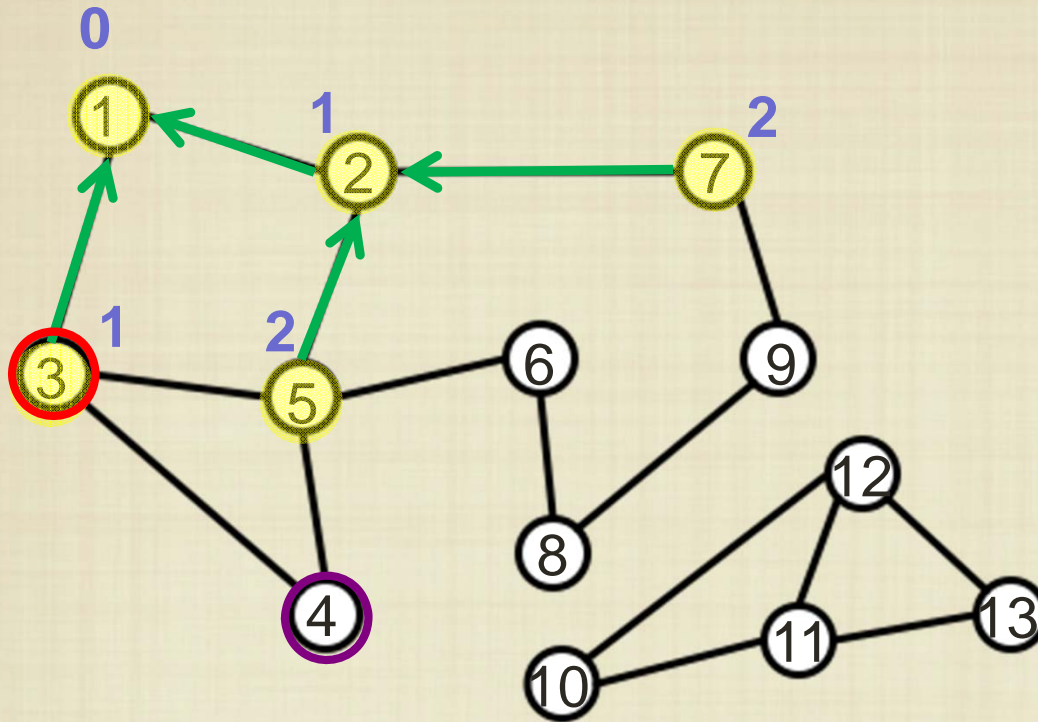
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pred	-1	1	1	-1	2	-1	2	-1	-1	-1	-1	-1	-1

vertex = 3
i = 1

queue = 5, 7

Breadth-First Search



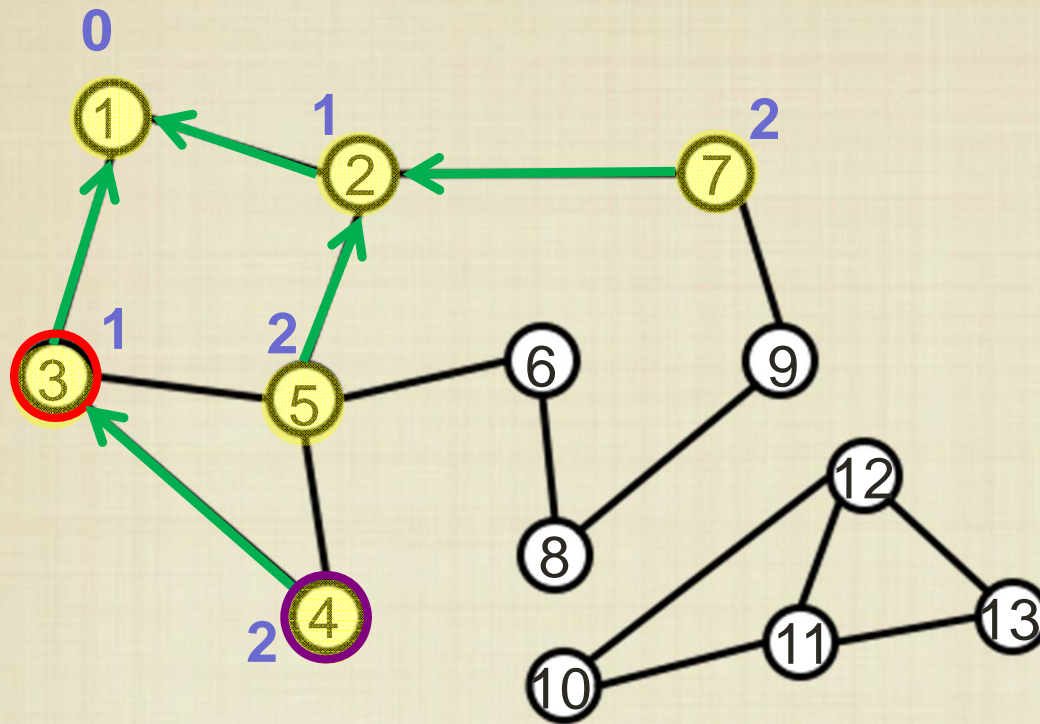
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pred	-1	1	1	-1	2	-1	2	-1	-1	-1	-1	-1	-1

vertex = 3
i = 4

queue = 5, 7

Breadth-First Search



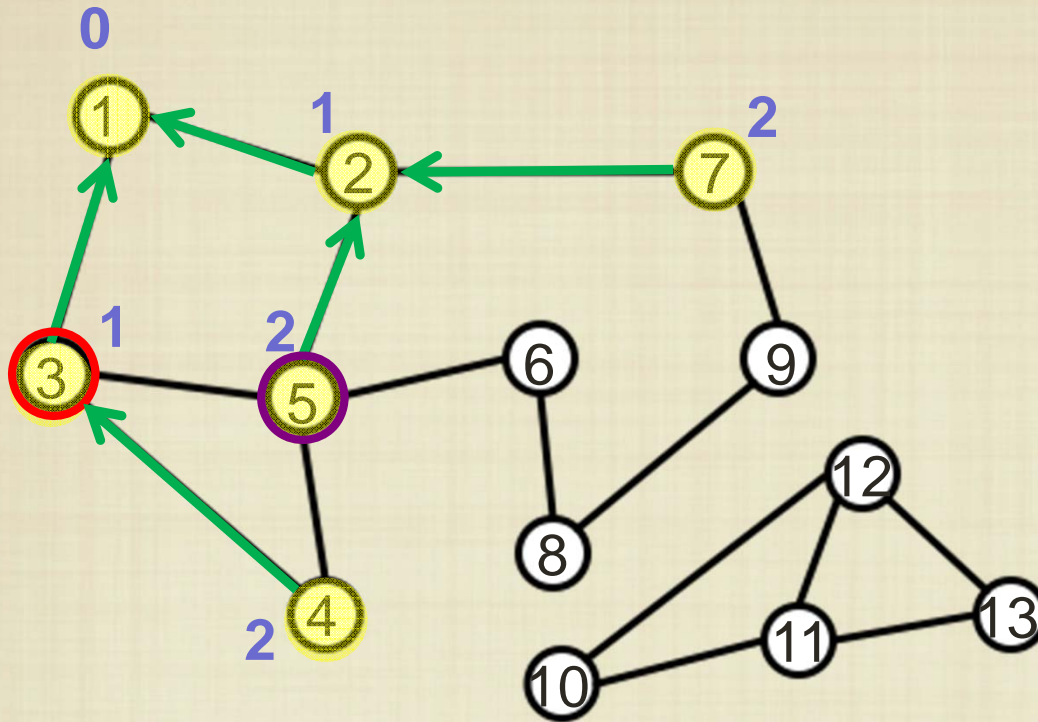
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pred	-1	1	1	3	2	-1	2	-1	-1	-1	-1	-1	-1

vertex = 3
i = 4

queue = 5, 7, 4

Breadth-First Search



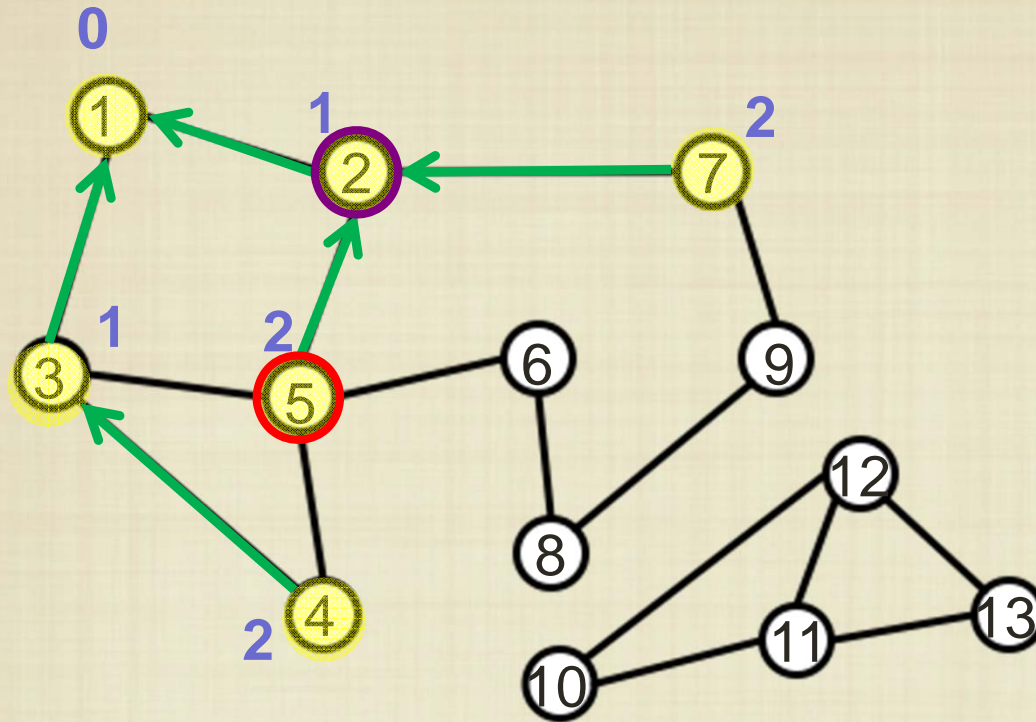
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pred	-1	1	1	3	2	-1	2	-1	-1	-1	-1	-1	-1

vertex = 3
i = 5

queue = 5, 7, 4

Breadth-First Search



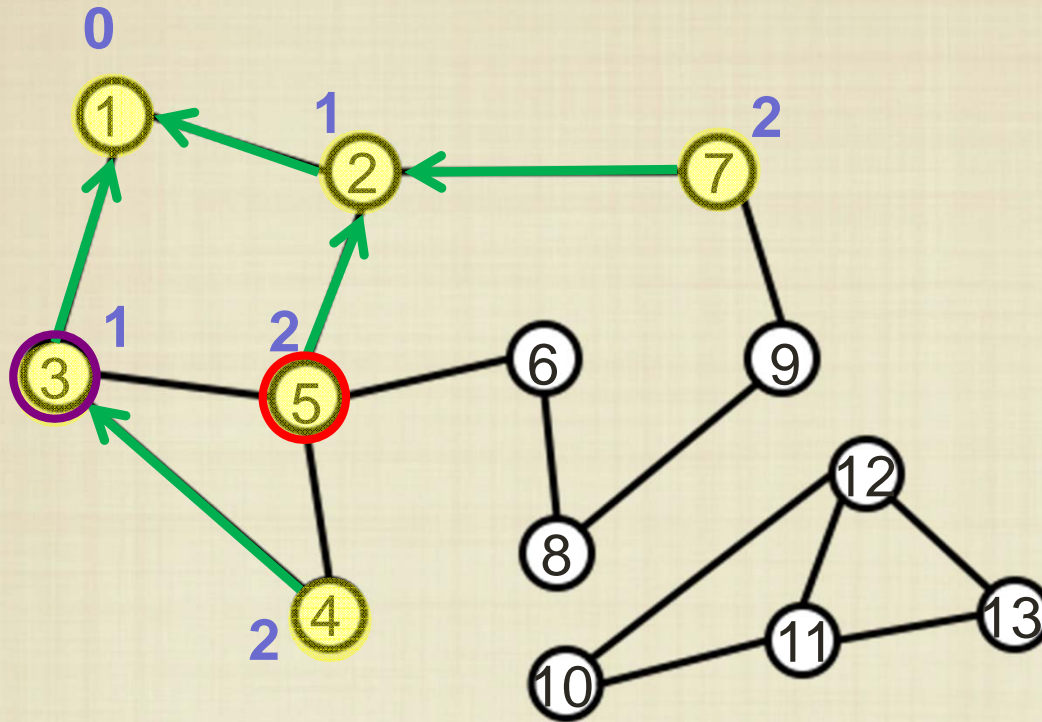
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pred	-1	1	1	3	2	-1	2	-1	-1	-1	-1	-1	-1

vertex = 5
i = 2

queue = 7, 4

Breadth-First Search



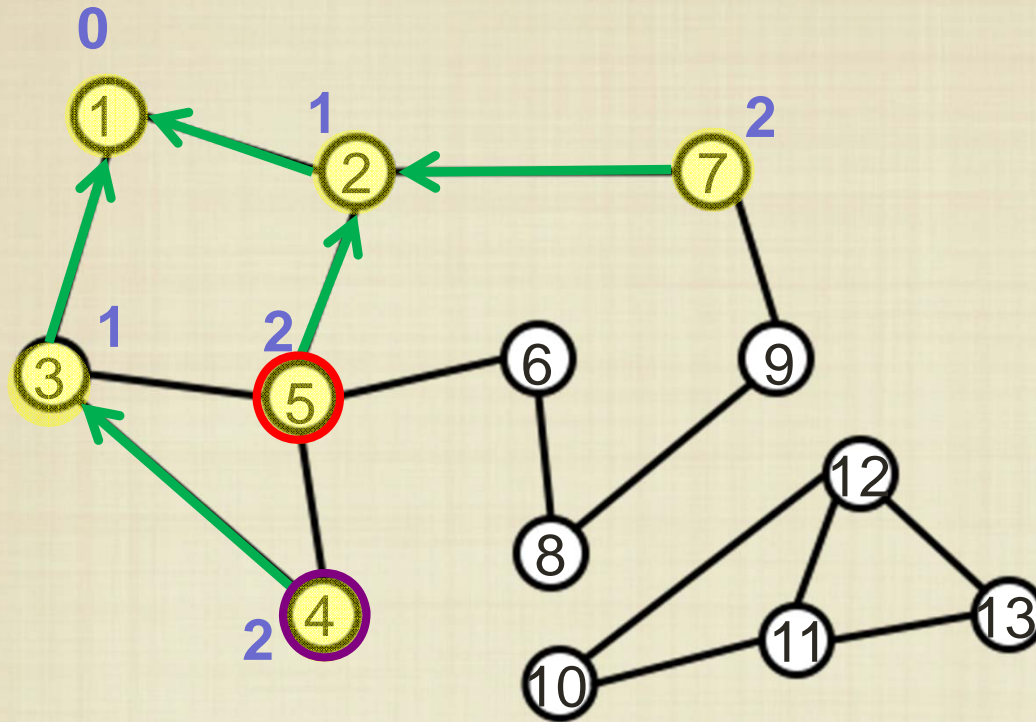
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vertex = 5
i = 3

queue = 7, 4

Breadth-First Search



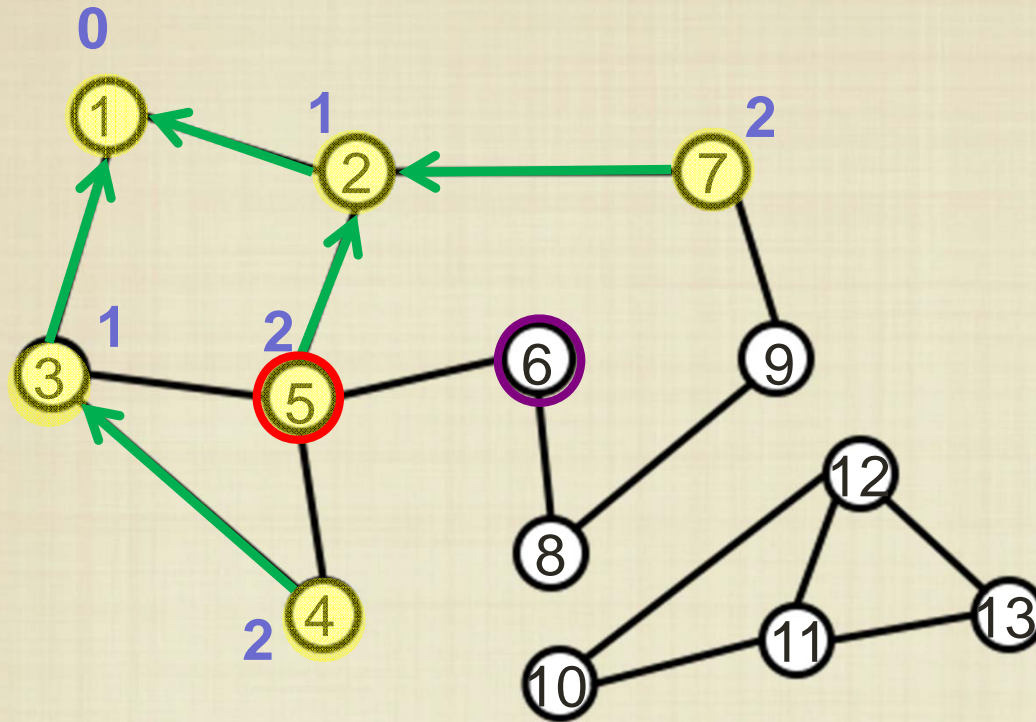
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distance	0	1	1	2	2	-1	2	-1	-1	-1	-1	-1	-1
pred	-1	1	1	3	2	-1	2	-1	-1	-1	-1	-1	-1

vertex = 5
i = 4

queue = 7, 4

Breadth-First Search



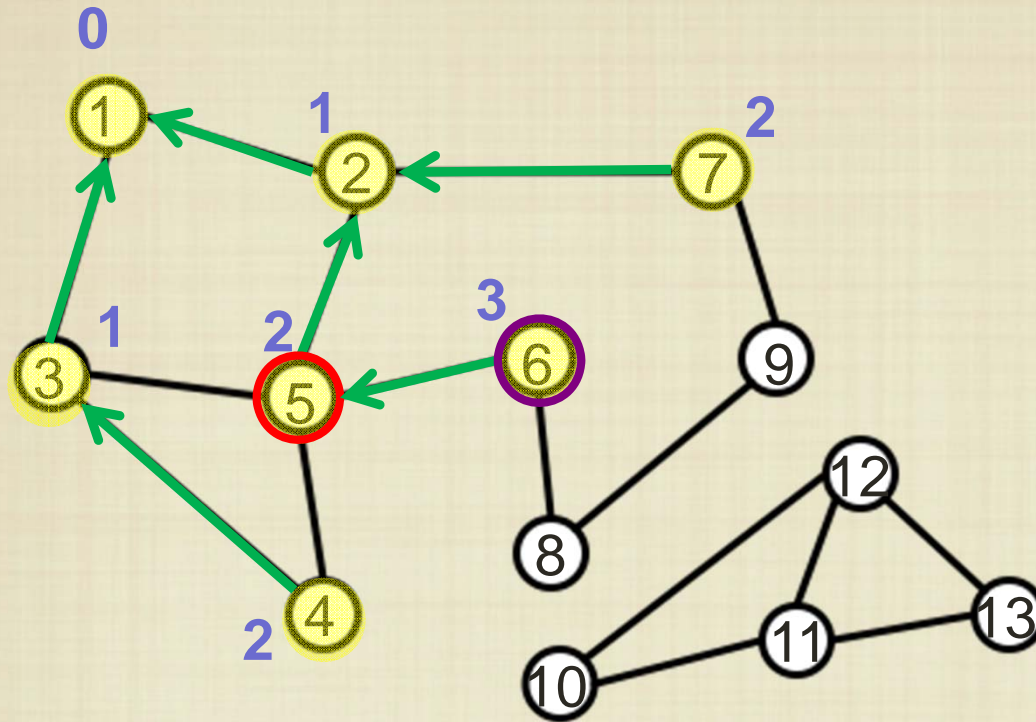
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traversed	True	True	True	True	True	False	True	False	False	False	False	False	False
distance	0	1	1	2	2	-1	2	-1	-1	-1	-1	-1	-1
pred	-1	1	1	3	2	-1	2	-1	-1	-1	-1	-1	-1

vertex = 5
i = 6

queue = 7, 4

Breadth-First Search



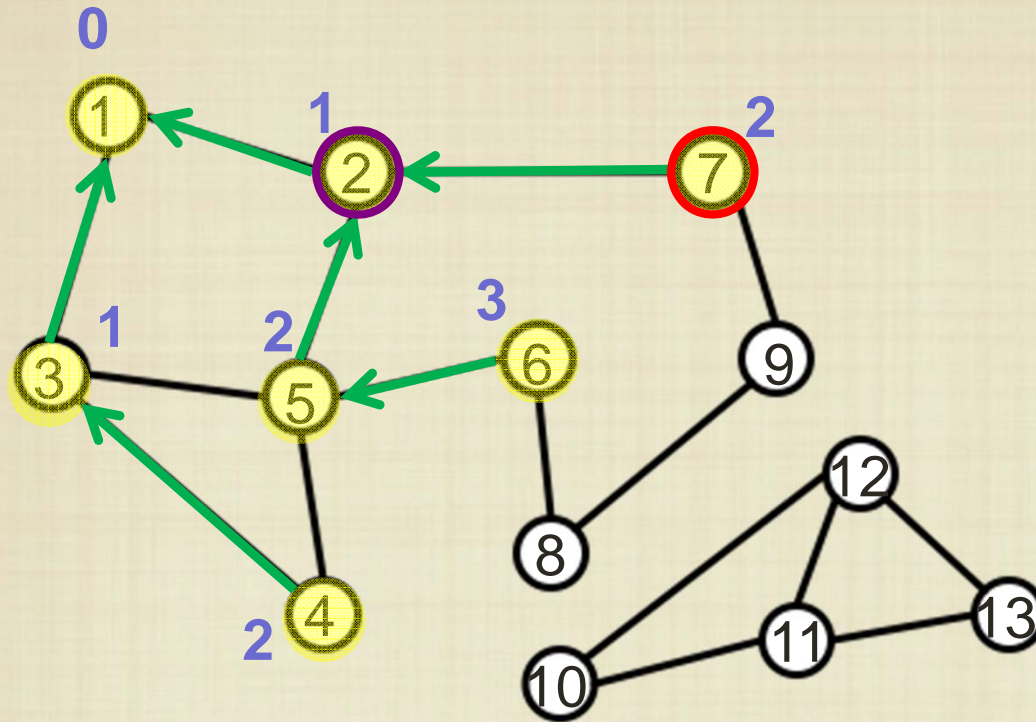
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distance	0	1	1	2	2	3	2	-1	-1	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	-1	-1	-1	-1	-1	-1

vertex = 5
i = 6

queue = 7, 4, 6

Breadth-First Search



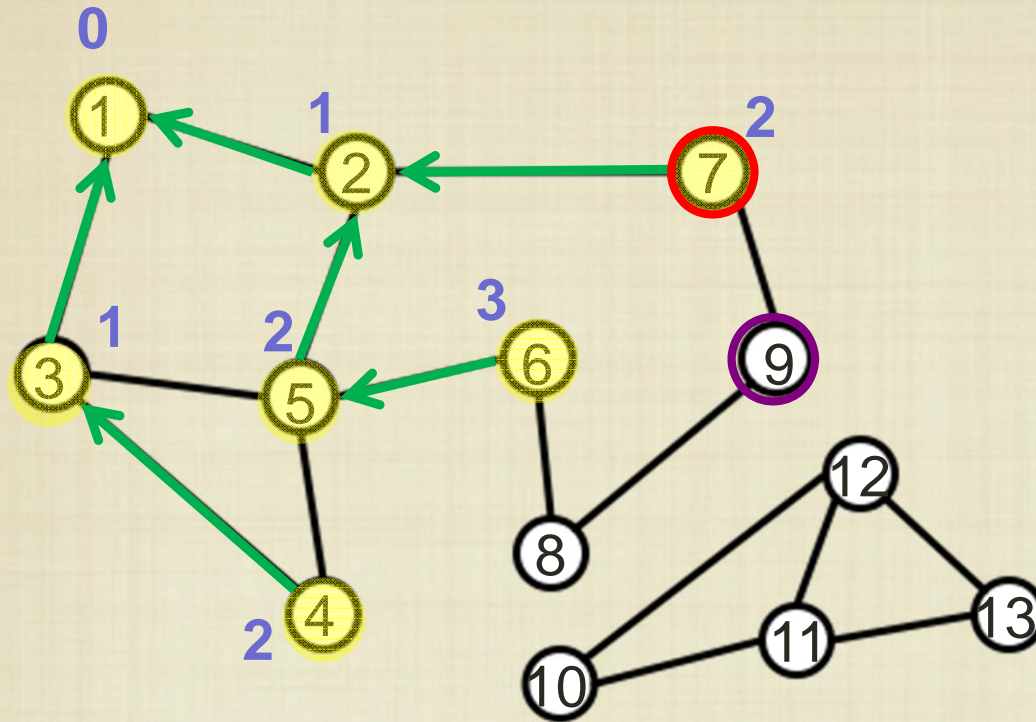
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distance	0	1	1	2	2	3	2	-1	-1	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	-1	-1	-1	-1	-1	-1

vertex = 7
i = 2

queue = 4, 6

Breadth-First Search



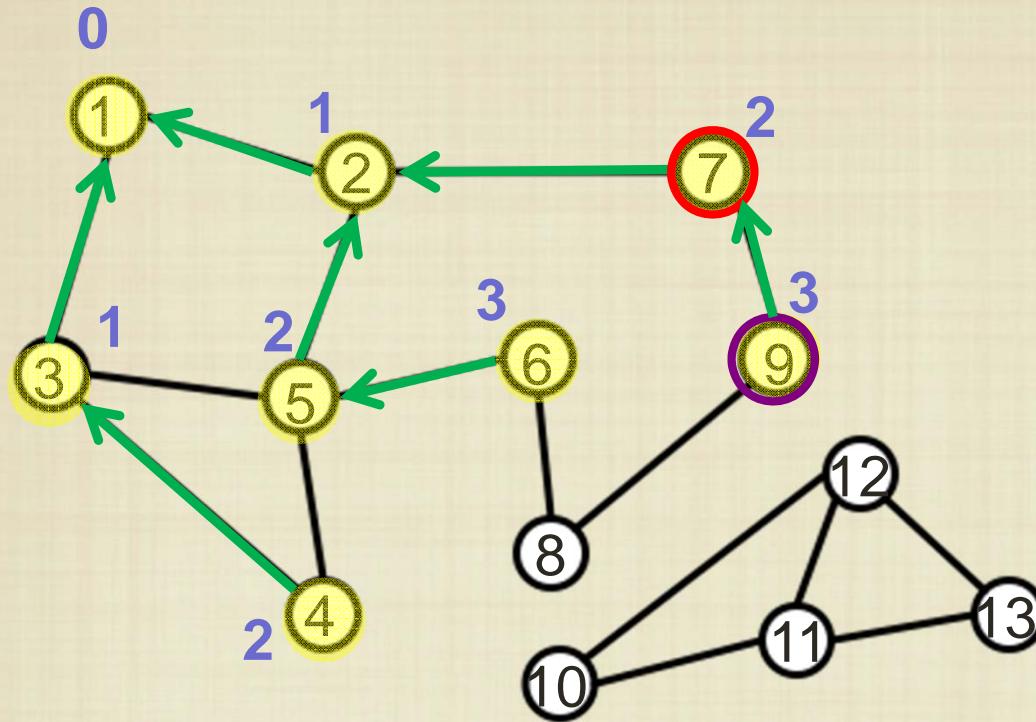
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distance	0	1	1	2	2	3	2	-1	-1	-1	-1	-1	-1
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vertex = 7
i = 9

queue = 4, 6

Breadth-First Search



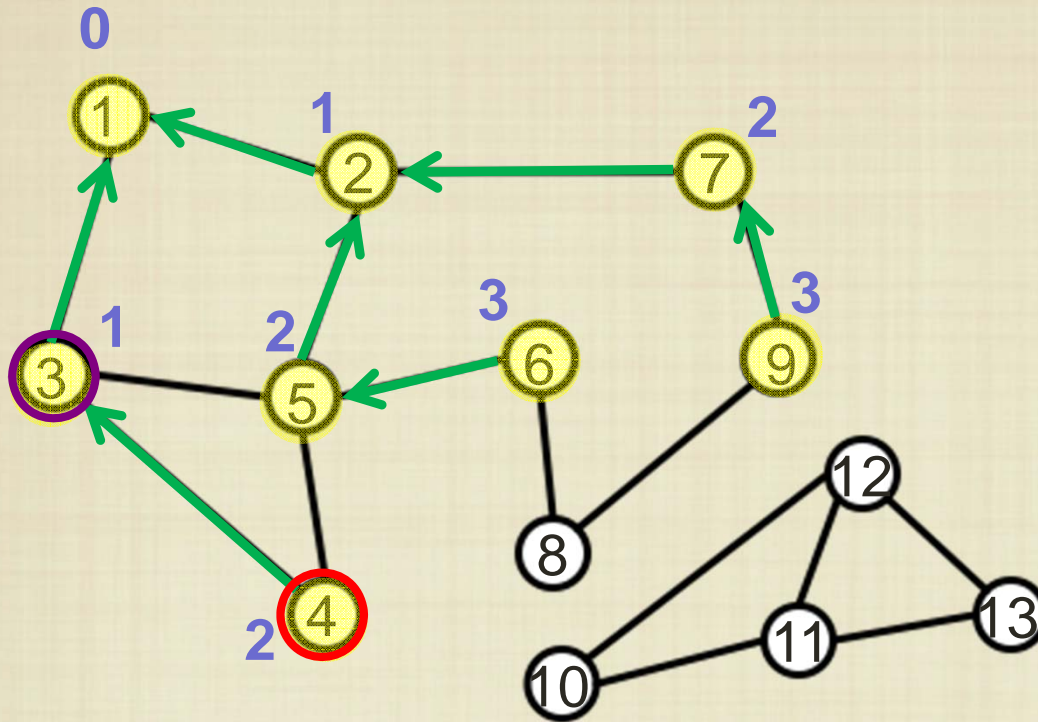
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pred	-1	1	1	3	5	5	2	-1	7	-1	-1	-1	-1

vertex = 7
i = 9

queue = 4, 6, 9

Breadth-First Search



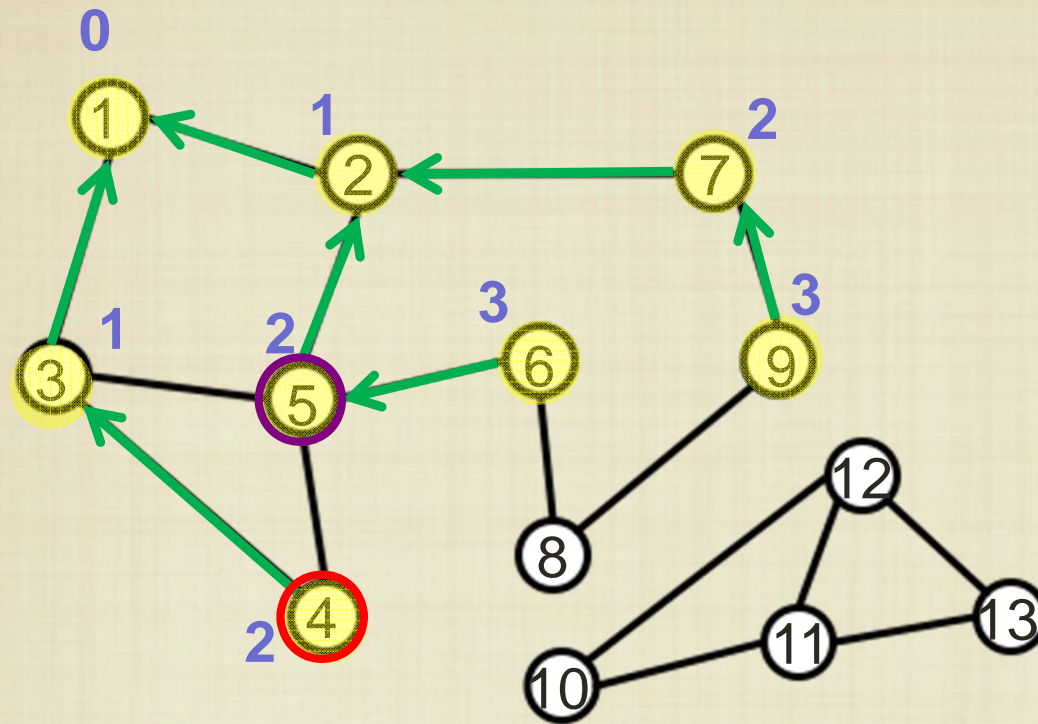
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 $\}$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	True	True	True	True	True	True	False	True	False	False	False	False
distance	0	1	1	2	2	3	2	-1	3	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	-1	7	-1	-1	-1	-1

vertex = 4
i = 3

queue = 6, 9

Breadth-First Search



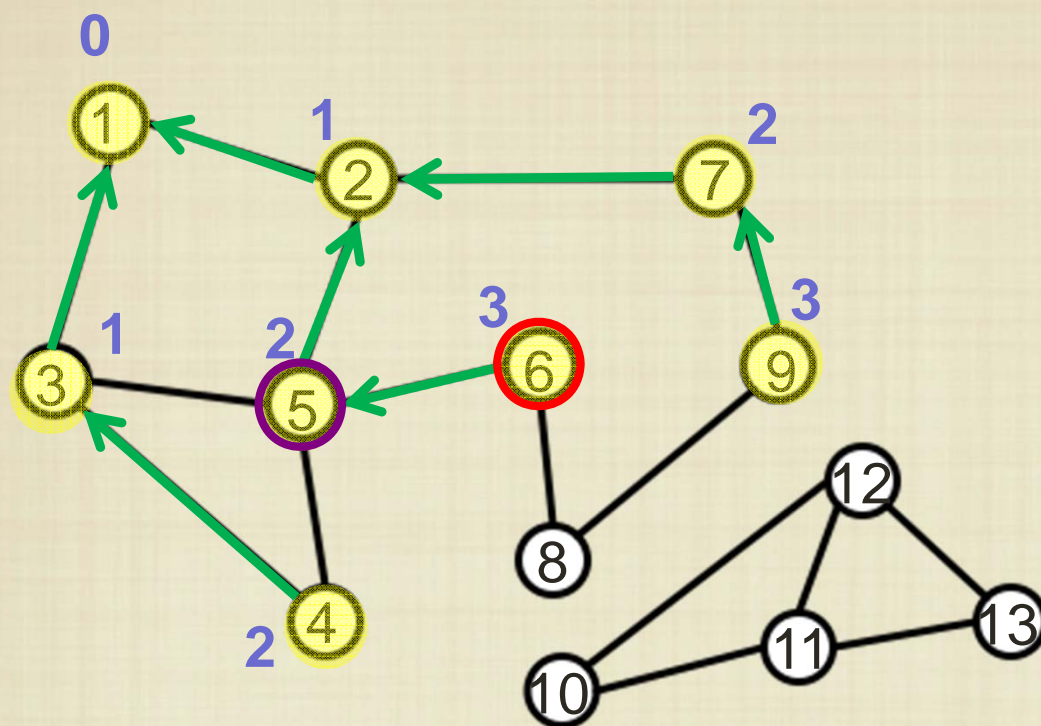
$G = \{$
 1:[2,3],
 2:[1,5,7],
 3:[1,4,5],
 4:[3,5],
 5:[2,3,4,6],
 6:[5,8],
 7:[2,9],
 8:[6,9],
 9:[7,8],
 10:[11, 12],
 11:[10,12,13],
 12:[10,11,13],
 13:[11,12]
 $\}$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	True	True	True	True	True	True	False	True	False	False	False	False
distance	0	1	1	2	2	3	2	-1	3	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	-1	7	-1	-1	-1	-1

vertex = 4
i = 5

queue = 6, 9

Breadth-First Search



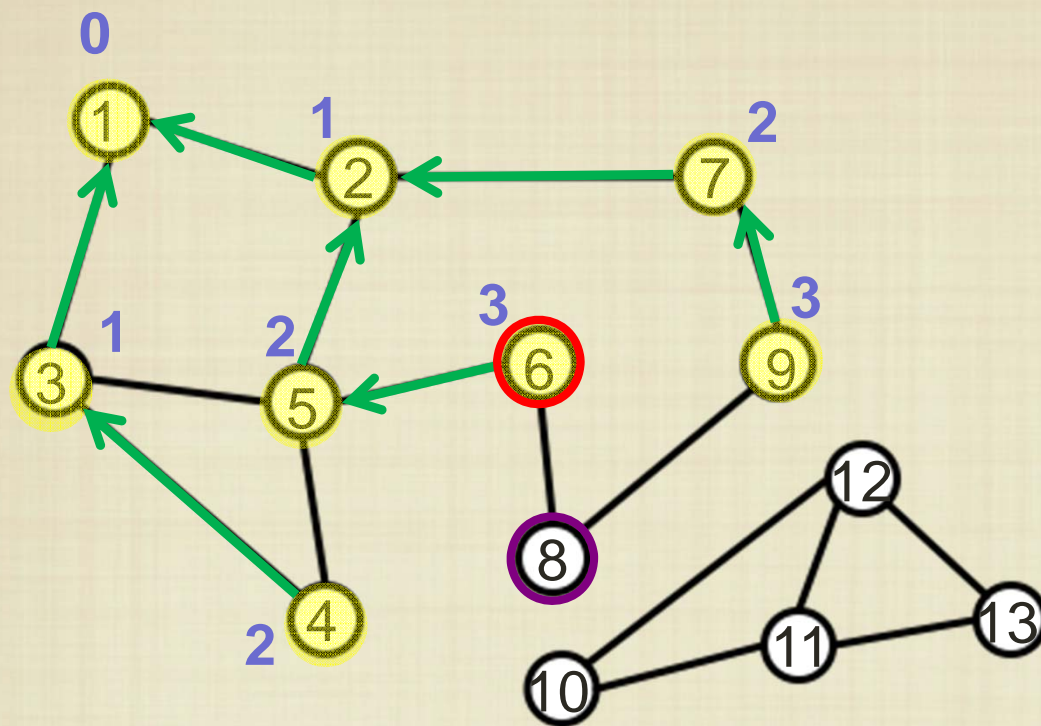
$G = \{$
 1:[2,3],
 2:[1,5,7],
 3:[1,4,5],
 4:[3,5],
 5:[2,3,4,6],
 6:[5,8],
 7:[2,9],
 8:[6,9],
 9:[7,8],
 10:[11, 12],
 11:[10,12,13],
 12:[10,11,13],
 13:[11,12]
 $\}$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	True	True	True	True	True	True	False	True	False	False	False	False
distance	0	1	1	2	2	3	2	-1	3	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	-1	7	-1	-1	-1	-1

vertex = 6
i = 5

queue = 9

Breadth-First Search



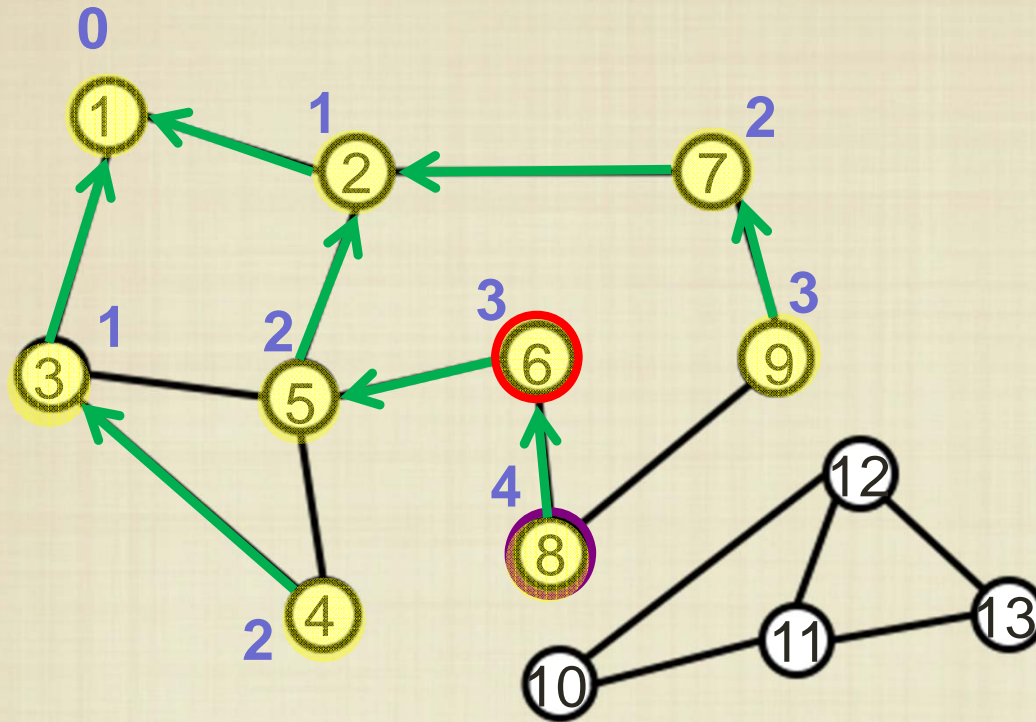
$G = \{$
 1:[2,3],
 2:[1,5,7],
 3:[1,4,5],
 4:[3,5],
 5:[2,3,4,6],
 6:[5,8],
 7:[2,9],
 8:[6,9],
 9:[7,8],
 10:[11, 12],
 11:[10,12,13],
 12:[10,11,13],
 13:[11,12]
 $\}$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	True	True	True	True	True	True	False	True	False	False	False	False
distance	0	1	1	2	2	3	2	-1	3	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	-1	7	-1	-1	-1	-1

vertex = 6
i = 8

queue = 9

Breadth-First Search



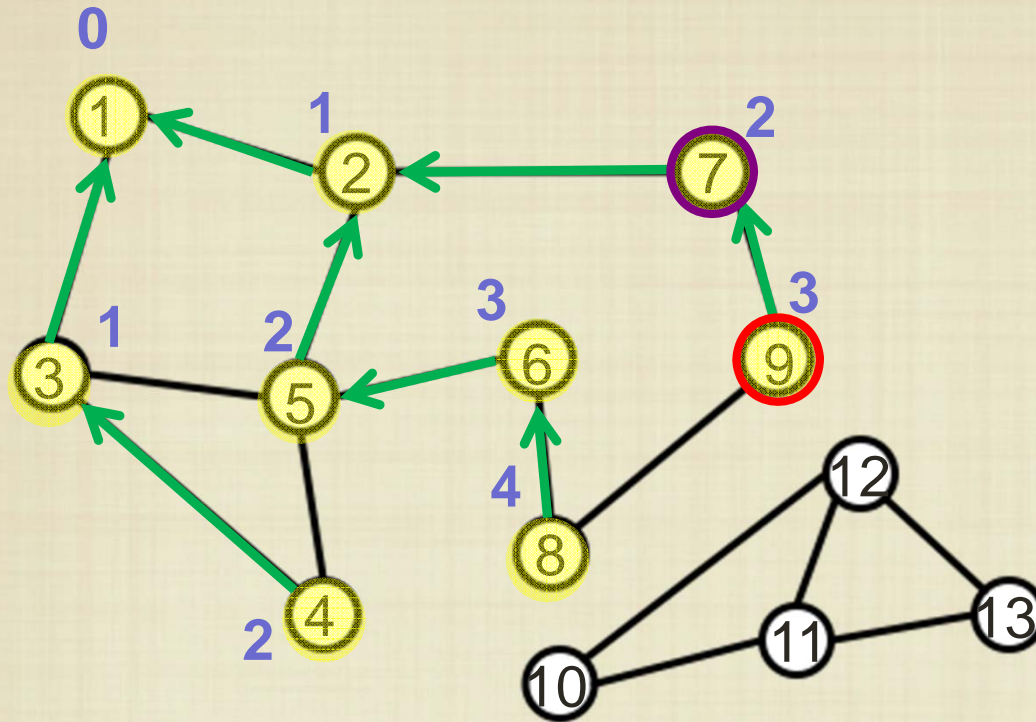
$G = \{$
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 2:[1,5,7],
 3:[1,4,5],
 4:[3,5],
 5:[2,3,4,6],
 6:[5,8],
 7:[2,9],
 8:[6,9],
 9:[7,8],
 10:[11, 12],
 11:[10,12,13],
 12:[10,11,13],
 13:[11,12]
 $\}$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	True	True	True	True	True	True	True	True	False	False	False	False
distance	0	1	1	2	2	3	2	4	3	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	6	7	-1	-1	-1	-1

vertex = 6
i = 8

queue = 9, 8

Breadth-First Search



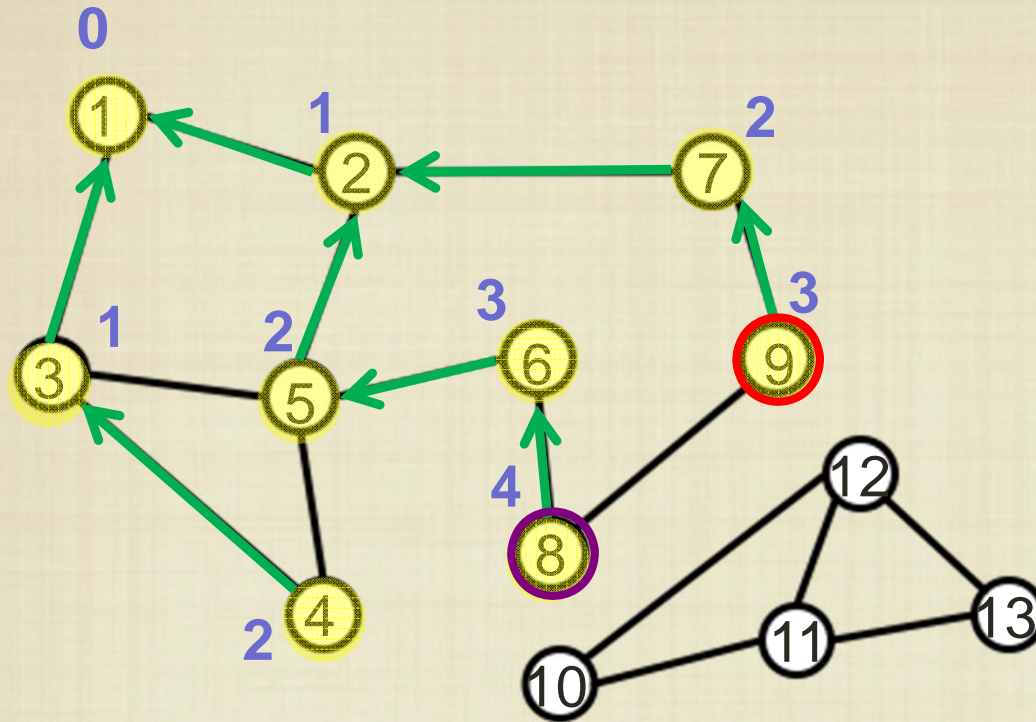
$G = \{$
 1:[2,3],
 2:[1,5,7],
 3:[1,4,5],
 4:[3,5],
 5:[2,3,4,6],
 6:[5,8],
 7:[2,9],
 8:[6,9],
 9:[7,8],
 10:[11, 12],
 11:[10,12,13],
 12:[10,11,13],
 13:[11,12]
 $\}$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	True	True	True	True	True	True	True	True	False	False	False	False
distance	0	1	1	2	2	3	2	4	3	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	6	7	-1	-1	-1	-1

vertex = 9
i = 7

queue = 8

Breadth-First Search



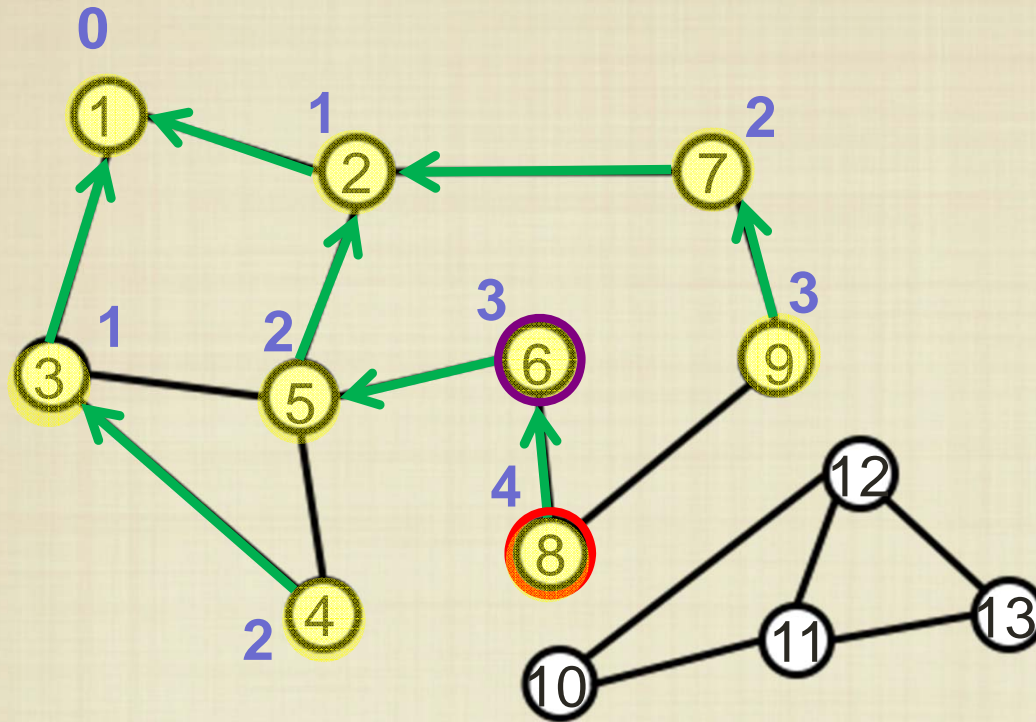
$G = \{$
 1:[2,3],
 2:[1,5,7],
 3:[1,4,5],
 4:[3,5],
 5:[2,3,4,6],
 6:[5,8],
 7:[2,9],
 8:[6,9],
 9:[7,8],
 10:[11, 12],
 11:[10,12,13],
 12:[10,11,13],
 13:[11,12]
 $\}$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	True	True	True	True	True	True	True	True	False	False	False	False
distance	0	1	1	2	2	3	2	4	3	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	6	7	-1	-1	-1	-1

vertex = 9
i = 8

queue = 8

Breadth-First Search


$$G = \{$$

- 1:[2,3],
- 2:[1,5,7],
- 3:[1,4,5],
- 4:[3,5],
- 5:[2,3,4,6],
- 6:[5,8],
- 7:[2,9],
- 8:[6,9],
- 9:[7,8],
- 10:[11, 12],
- 11:[10,12,13],
- 12:[10,11,13],
- 13:[11,12]

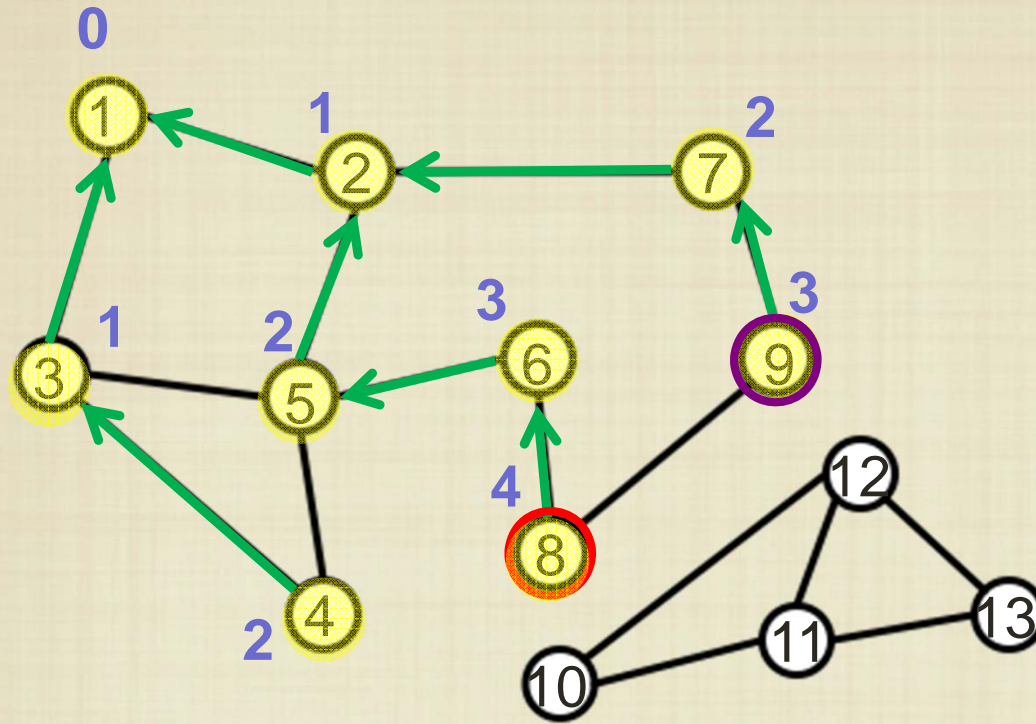
$$\}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	True	True	True	True	True	True	True	True	False	False	False	False
distance	0	1	1	2	2	3	2	4	3	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	6	7	-1	-1-	-1	-1

vertex = 8
i = 6

queue =

Breadth-First Search


$$G = \{$$

- 1:[2,3],
- 2:[1,5,7],
- 3:[1,4,5],
- 4:[3,5],
- 5:[2,3,4,6],
- 6:[5,8],
- 7:[2,9],
- 8:[6,9],
- 9:[7,8],
- 10:[11, 12],
- 11:[10,12,13],
- 12:[10,11,13],
- 13:[11,12]

$$\}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
traversed	True	True	True	True	True	True	True	True	True	False	False	False	False
distance	0	1	1	2	2	3	2	4	3	-1	-1	-1	-1
pred	-1	1	1	3	5	5	2	6	7	-1	-1	-1	-1

vertex = 8
i = 9

queue =

Breadth-First Search

Runtime:

- Each vertex is enqueued at most once, and therefore dequeued at most once
 - We need to examine each edge once to update distances
- ⇒ Runtime $O(n+m)$ where $n=|V|$ is the number of vertices in G , and $m=|E|$ is the number of edges in G
- ⇒ The runtime is linear in the size of the graph